Multiple attribute strategic weight manipulation with minimum cost in a group decision making context with interval attribute weights information

Yating Liu, Yucheng Dong, Haiming Liang, Francisco Chiclana, Enrique Herrera-Viedma

***Abstract*—In multiple attribute decision making (MADM), strategic weight manipulation is understood as a deliberate manipulation of attribute weights setting to achieve a desired ranking of alternatives. In this paper, we study the strategic weight manipulation in a group decision making context with interval attribute weights information. In group decision making, the revision of the decision makers' original attribute weights information implies a cost (the difference between the original information and the revised one). Driven by a desire to minimize the cost, we propose the minimum cost strategic weight manipulation model, which is achieved via optimization approaches, with the 0-1 mixed linear programming model being proved appropriate in this context. Meanwhile, some desired properties to manipulate a strategic attribute weight based on the ranking range under interval attribute weights information are proposed. Finally, numerical analysis and simulation experiments are provided with a two-fold aim: (1) to verify the validity of the proposed models, and (2) to show the effects of interval attribute weights information and the unit cost, respectively, on the cost to manipulate strategic weights in the MADM in a group decision context.**

***Keywords*—multiple attribute decision making, strategic weight manipulation, minimum cost, interval attribute weights information**

I. INTRODUCTION

Multiple attribute decision making (MADM) aims to obtain a ranking of alternatives based on their evaluation information regarding multiple attributes. MADM has received increasing attention in decision analysis [22, 37, 45, 46], and it has been applied in a wide range of fields [5, 8, 19, 33].

Attribute weights play an important role in the resolution of MADM problems [28, 29]. Until now, there exist many approaches in the specialized literature on how to obtain the attribute weights in MADM. The existing approaches can mainly be divided into three categories [12]:

(1) *The subjective approach* obtains the attribute weights according to the decision makers' subjective preference information on the set of attributes. For example, Doyle et al. [15] proposed a direct rating (DR) method and a point allocation (PA) method; Barron and Barrett [1] investigated three rank-ordered methods; while Roberts and Goodwin [32] provided a rank order distribution (ROD) approach.

(2) *The objective approach* determines the attribute weights by using objective decision matrix information and the entropy method [47]; or a TOPSIS-based method [48]; or some other mathematical programming based method [7, 35].

(3) *The integrated approach* obtains the attribute weights according to both the decision makers' subjective preference information and the objective decision matrix information. For example, Cook and Kress [10] proposed a preference-aggregation model; while Fan et al. [16], Horsky and Rao [20], Pekelman and Sen [30] constructed optimization-based models.

Strategic manipulation or non-cooperative behavior in decision making describe those situations in which some decision makers dishonestly express opinions to enhance the chances of obtaining their most preferred alternatives. Strategic manipulation is a common phenomenon and has been analyzed in depth in different decision contexts. For example, Pelta et al. [31] and Yager [42, 43] have proposed aggregation approaches to defend against the strategic manipulation in group decision making (GDM); whereas Dong et al. [13], Palomares et al. [26], and Xu et al. [41] have investigated how to detect and manage a series of non-cooperative behaviors in GDM consensus reaching processes from different perspectives.

As mentioned above, approaches to set attribute weights have been investigated intensively; however, in these approaches decision makers are assumed to be honest when expressing their preferences regarding attribute weights. Recently, Dong et al. [12] proposed the concept of strategic weight manipulation, in which a decision maker can be dishonest in the sense of setting attribute weights strategically to obtain his/her desired ranking of alternatives. Although this

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work by Dong et al. is useful in MADM, there still exist issues that needs to be addressed:

(1) In [12] the strategic weight manipulation was investigated in an individual decision making context. However, the increasing complexity of decision environments means that many practical decisions involve multiple decision makers. Additionally, the strategic weight manipulation investigated in [12] assumed no constraints on the weights and consequently the strategic attribute weights could be set freely as anyone of the domain values. However, decision makers often will present some attribute weights information [6, 21, 23, 27] and thus some attributes weights information are partially known or subject to certain constraints. So, it is necessary to investigate the strategic weight manipulation in a group decision context in which attribute weights information are partially known.

(2) When decision makers provide partially attribute weights information in a group decision context, it is more challenging for a manipulator to strategically set attribute weights because some decision makers may be reluctant to change their original attribute weight preferences. As a result, the manipulator needs to assume some cost for decision makers to revise their original attribute weight preferences. Driven by a desire to minimize the cost, it is necessary to investigate the strategic weight manipulation with minimum cost.

In order to address these two issues, this paper proposes the strategic weight manipulation with minimum cost in a group decision making context with interval attribute weights information. The proposed methodology to achieve this consists of the following main steps:

- Attribute weights are considered partially known, and they are described by numerical intervals, i.e. interval attribute weights information is assumed. Additionally, multiple decision makers are assumed to be involved in the strategic weight manipulation. Following these assumption, this paper develops a new strategic weight manipulation model in a group decision context with interval attribute weights information.
- A minimum cost model is developed to strategically set the attribute weights, by revising the decision makers' original preferences of attribute weights to obtain a desired ranking of alternatives. Meanwhile, some desired properties with zero cost for manipulating strategic attribute weights are explored. Simulation experiments with real data are provided to show the effects of the interval attribute weights information and the unit cost, respectively, in the cost to manipulate strategic weights in the MADM in a group context.

The remainder of this paper is organized as follows: Section 2 introduces some basic concepts regarding the MADM. Mixed 0-1 linear programming models to set a multiple attribute strategic weight vector with minimum cost are constructed in section 3. Section 4 presents numerical analysis and simulation experiments to justify the proposal put forward in the paper. Concluding remarks and future research agenda are included in Section 5.

II. Preliminaries

This section introduces some basic knowledge regarding MADM and attribute weights.

A. *Classical MADM problem*

A classical MADM problem can be described as follows: let $X=\{x_1,...,x_n\}$ be a finite set of alternatives, $A=\{a_1,...,a_m\}$ a set of predefined attributes, and $\mathbf{w}=(w_1,w_2,...,w_m)^T$ the weight vector of the attributes, where $w_j\geq 0$ and $\sum_{j=1}^{m} w_j=1$. Let $V=[v_{ij}]_{n\times m}$ be the decision matrix, where v_{ij} denotes the attribute value associated with alternative $x_i\in X$ and attribute $a_j\in A$. The resolution process of a MADM problem includes, generally, two steps:

(1) *Normalization phase.* Attributes are split into two categories: benefit attributes and cost attributes. The decision matrix $V=[v_{ij}]_{n\times m}$ is transformed into a normalized decision matrix $\bar{V}=[\bar{v}_{ij}]_{n\times m}$, where

$$\bar{v}_{ij}=\frac{v_{ij}-\min_i(v_{ij})}{\max_i(v_{ij})-\min_i(v_{ij})} \tag{1}$$

if $a_j\in A$ is a benefit attribute, while

$$\bar{v}_{ij}=\frac{\max_i(v_{ij})-v_{ij}}{\max_i(v_{ij})-\min_i(v_{ij})} \tag{2}$$

if $a_j\in A$ is a cost attribute.

(2) *Ranking of alternatives.* Alternatives are ranked by associating them with an evaluation value $D_{\mathbf{w}}(x_i)$, which is computed by a decision function F that assigns an overall evaluation to each alternative, i.e. $D_{\mathbf{w}}(x_i)=F_{\mathbf{w}}(\bar{v}_{i1},\bar{v}_{i2},..., \bar{v}_{im})$, with $\mathbf{w}=(w_1,w_2,...,w_m)^T$ being the attribute weight vector. It is worth mentioning at this point that the alternatives' overall evaluation is frequently derived by fusing their attributes normalized decision values, i.e. by using as functions F an aggregation operator such as the weighted average (WA) or the ordered weighted average (OWA) operators [38, 44], which would result, respectively, in:

$$D_{\mathbf{w}}(x_i)=WA_{\mathbf{w}}(\bar{v}_{i1},\bar{v}_{i2},...,\bar{v}_{im})=\sum_{j=1}^{m} w_j\bar{v}_{ij} \tag{3}$$

$$D_{\mathbf{w}}(x_i)=OWA_{\mathbf{w}}(\bar{v}_{i(1)},\bar{v}_{i(2)},...,\bar{v}_{i(m)})=\sum_{j=1}^{m} w_j\bar{v}_{i(j)} \tag{4}$$

where $\bar{v}_{i(j)}$ is the j th largest value in $\{\bar{v}_{i1},\bar{v}_{i2},...,\bar{v}_{im}\}$.

There exist various approaches to rank alternatives. However, as this paper is a continuation of the study presented in [12], the ranking approach used there is also employed here: let $Q_{h,w}=\{x_i \mid D_w(x_i)>D_w(x_h),\ i=1,2,...,n\}$ be the set of the alternatives whose decision evaluation value is greater than that of the alternative x_h, and $|Q_{h,w}|$ its cardinality. Then, the ranking position of the alternative x_h is

$$r_w(x_h) = |Q_{h,w}| + 1$$
(5)

B. Research problem: Attribute weights in a group decision context with interval attribute weights information

As mentioned in Dong et al. [12], the setting of attribute weights has an important effect on the ranking of alternatives. Thus, a manipulator may strategically set the attribute weights to attain his/her desired ranking in the MADM.

Generally, in real-life MADM problems, the decision matrix $V = [v_{ij}]_{n\times m}$ is considered as representing objective information, with the attribute weights being set by one or more decision makers.

We make the following assumption (1):

(1) Let $E = \{e_1, e_2, ..., e_l\}$ be a set of decision makers and let $\mathbf{w^k} = (w_1^k, w_2^k, ..., w_m^k)^T$ be the weight vector of the attributes associated with the decision maker $e_k \in E$, where $w_j^k \geq 0$ and $\sum_{j=1}^{m} w_j^k = 1$. The attribute weight vector is determined as the average of all decision makers' corresponding attribute weight vectors:

$$w_j = \frac{\sum_{k=1}^{l} w_j^k}{l} \tag{6}$$

In MADM problems, because of time pressure or limited expertise, some decision makers might not be able to provide precise attribute weights but incomplete attribute weights instead [6, 21, 23, 26], i.e. some information on attributes weights may be unknown or represented as interval values. Usually, the basic forms of incomplete attribute weights include weak ranking, strict ranking, ranking multiples, interval form, ranking differences and bounded (see [21, 23, 26]). In this paper, we consider interval attribute weights, i.e. the attribute weights are in some numerical intervals.

Then, we make the following assumption (2):

(2) The attribute weight vector $\mathbf{w^k} = (w_1^k, w_2^k, ..., w_m^k)^T$, associated with the decision maker $e_k \in E$, is an interval weight vector, i.e.

$$w_j^k = [I_j^{k,-}, I_j^{k,+}] \tag{7}$$

where $0 \leq I_j^{k,-} \leq I_j^{k,+} \leq 1$. When conditions $\sum_{i=1}^{m} I_i^{k,+} - \max_j (I_j^{k,+} - I_j^{k,-}) \geq 0$ and $\sum_{i=1}^{m} I_i^{k,-} + \max_j (I_j^{k,+} - I_j^{k,-}) \leq 1$ are verified, $\mathbf{w^k}$ is said to be a normalized interval weight vector [37]. These conditions guarantee that there exists a weight vector $\mathbf{w} = (w_1, w_2, ..., w_m)^T$ such that $\sum_{j=1}^{m} w_j = 1$ and $I_j^{k,-} \leq w_j \leq I_j^{k,+} (\forall j)$.

When the decision makers have interval information of attribute weights, setting strategic attribute weights carries a cost as the original attribute weights information has to be revised, i.e. modified. Inspired by the classical minimum cost model [2, 3], in this paper we study the multiple attribute strategic weight manipulation with minimum cost in a GDM context with interval attribute weights information. Minimum cost strategic weight manipulation will be formulated and discussed in the next section.

III. Multiple Attribute Strategic Weight Manipulation with Minimum Cost

This section contains: (1) the strategic weight manipulation with minimum cost in MADM, (ii) an approach based on the mixed 0-1 linear programming to obtain its optimal solution, and (iii) some desired properties.

A. Basic ideas and model

In this subsection, we introduce some basic ideas and construct an optimization-based model with minimum cost to find out the manipulator's strategic weight vector to obtain his/her desired ranking of alternative(s).

In this study, without loss of generality, we assume that the manipulator wants to manipulate the alternatives $\{x_1, x_2, ..., x_g\}$, where $g \geq 1$ and $g \in N^+$, to which the attribute weight vector $\mathbf{w} = (w_1, w_2, ..., w_m)^T$ is to be strategically set.

Let $\mathbf{w^k} = (w_1^k, w_2^k, ..., w_m^k)^T$ be the original normalized interval attribute weights vector associated with the decision maker e_k. In order to strategically set the attribute weight vector $\mathbf{w} = (w_1, w_2, ..., w_m)^T$, the manipulator hopes that the decision makers can revise their original interval information regarding attribute weight vectors. Let us denote by $\overline{\mathbf{w^k}} = (\overline{w_1^k}, \overline{w_2^k}, ..., \overline{w_m^k})^T$ the revised attribute weight vector associated with the decision maker e_k, where $\sum_{j=1}^{m} \overline{w_j^k} = 1$ and $0 \leq \overline{w_j^k} \leq 1$. The difference between the original and the revised attribute weight vector associated with the decision maker e_k can be measured by

$$d\left(\boldsymbol{w^k}, \overline{\boldsymbol{w^k}}\right) = \sum_{j=1}^{m} d\left(w_j^k, \overline{w_j^k}\right)$$
(8)

where

$$d\left(w_j^k, \overline{w_j^k}\right) = \begin{cases} I_j^{k,-} - \overline{w_j^k}, & 0 \leq \overline{w_j^k} < I_j^{k,-}, \\ 0, & I_j^{k,-} \leq \overline{w_j^k} \leq I_j^{k,+}, \\ \overline{w_j^k} - I_j^{k,+}, & I_j^{k,+} < \overline{w_j^k} \leq 1, \end{cases} \tag{9}$$

Motivated by the minimum cost model, setting strategic attribute weights means the manipulator needs to take some cost for the decision makers to revise their original interval attribute weights information. Let f_k be the unit cost to revise the decision maker e_k's attribute weight. The unit cost is a basic concept of minimum cost GDM models [17, 18, 24, 51, 53], and refers to the cost for the decision makers adjusting the unit opinions. Usually, the unit cost can be measured by money, time and so on. In practical GDM context, the manipulator often assumes the cost to persuade the decision makers in changing their opinions, and the finalized measurement for the cost is determined by the specified decision making problem. Generally, the greater the distance of experts changing their opinion, the greater the cost. Thus, the cost function of revising the decision maker e_k's attribute weight can be defined as the product of the unit cost and distance of opinions changing, $f_k d\left(\boldsymbol{w}^k, \overline{\boldsymbol{w}^k}\right)$. Thus, the cost function of revising all the decision makers' attribute weights can be denoted as Eq. (10)

$$\sum_{k=1}^{l} f_k d\left(\boldsymbol{w}^k, \overline{\boldsymbol{w}^k}\right) = \sum_{k=1}^{l}\sum_{j=1}^{m} f_k d\left(w_j^k, \overline{w_j^k}\right) \quad (10)$$

It is assumed that the manipulator aims to minimize the cost, that is

$$\min \sum_{k=1}^{l}\sum_{j=1}^{m} f_k d\left(w_j^k, \overline{w_j^k}\right) \quad (11)$$

Meanwhile, following Eq. (6), the attribute weight vector strategically set by the manipulator is determined as follows:

$$w_j = \frac{\sum_{k=1}^{l} \overline{w_j^k}}{l} \quad (12)$$

Moreover, we assume the manipulator's desired ranking of the alternatives $\{x_1, x_2, ..., x_g\}$ is $\{r^*(x_1), r^*(x_2), ..., r^*(x_g)\}$, i.e.

$$r_w(x_h) = r^*(x_h) \quad (h = 1, 2, ..., g) \quad (13)$$

Based on Eqs. (8-13), we construct the minimum cost strategic weight manipulation (MCSWM) model to set the strategic weight vector as follows:

$$\begin{cases} \min \sum_{k=1}^{l}\sum_{j=1}^{m} f_k d\left(w_j^k, \overline{w_j^k}\right) \\ s.t. \begin{cases} r_w(x_h) = r^*(x_h), \quad (h = 1, 2, ..., g) \\ \mathbf{w} = (w_1, w_2, ..., w_m)^T \\ w_j = \dfrac{\sum_{k=1}^{l} \overline{w_j^k}}{l}, \quad (j = 1, 2, ..., m) \\ \sum_{j=1}^{m} \overline{w_j^k} = 1, \quad (k = 1, 2, ..., l) \\ 0 \le \overline{w_j^k} \le 1, \quad (j = 1, 2, ..., m) \\ w_j^k = [I_j^{k,-}, I_j^{k,+}], \quad (k = 1, 2, ..., l; j = 1, 2, ..., m) \end{cases} \end{cases} \quad (14)$$

where $\overline{w_j^k}$, $(k = 1, 2, ..., l; j = 1, 2, ..., m)$ are the decision variables.

B. Solving the minimum cost strategic weight manipulation model via mixed 0-1 linear programming

In this subsection, we continue to use a mixed 0-1 linear programming methodology to obtain the optimal solution to the MCSWM (model (14)).

In order to transform model (14) into a mixed 0-1 linear programming, binary variable $y_{ih} \in \{0,1\}$ and a large enough number M are introduced. We have the following results:

(1) $x_i \succ x_h$ if and only if $y_{ih} = 1$ under the conditions: $D_w(x_i) > D_w(x_h) - (1 - y_{ih})M$ and $D_w(x_i) \le D_w(x_h) + y_i M$.

(2) $x_i \precsim x_h$ if and only if $y_{ih} = 0$ under the conditions: $D_w(x_i) \le D_w(x_h) + y_{ih}M$ and $D_w(x_i) > D_w(x_h) - (1 - y_{ih})M$.

The following Lemmas are proposed:

Lemma 1: For decision function with F the WA operator as per Eq. (3), if there exists $\mathbf{w}^* = (w_1^*, w_2^*, ..., w_m^*)^T$ satisfying constraint conditions (15)-(23)

$$\begin{cases} \sum_{j=1}^{m} w_j^* \bar{v}_{ij} > \sum_{j=1}^{m} w_j^* \bar{v}_{hj} - (1 - y_{ih})M, \quad (i = 1, 2, ..., n; h = 1, 2, ..., g) & (15) \\ \sum_{j=1}^{m} w_j^* \bar{v}_{ij} \le \sum_{j=1}^{m} w_j^* \bar{v}_{hj} + y_{ih}M, \quad (i = 1, 2, ..., n; h = 1, 2, ..., g) & (16) \\ y_{ih} = 1 \ \ or \ \ 0, \quad (h = 1, 2, ..., g) & (17) \\ \sum_{i=1}^{n} y_{ih} + 1 = r^*(x_h), \quad (i = 1, 2, ..., n; h = 1, 2, ..., g) & (18) \\ w_j^* = \dfrac{\sum_{k=1}^{l} \overline{w_j^{*,k}}}{l}, \quad (j = 1, 2, ..., m) & (19) \\ \sum_{j=1}^{m} \overline{w_j^{*,k}} = 1, \quad (k = 1, 2, ..., l) & (20) \\ 0 \le \overline{w_j^{*,k}} \le 1, \quad (j = 1, 2, ..., m) & (21) \\ w_j^k = [I_j^{k,-}, I_j^{k,+}], \quad (k = 1, 2, ..., l; j = 1, 2, ..., m) & (22) \\ d(\overline{w_j^{*,k}}, w_j^k) = \begin{cases} I_j^{k,-} - \overline{w_j^{*,k}}, & 0 \le \overline{w_j^{*,k}} \le I_j^{k,-}, \\ 0, & I_j^{k,-} < \overline{w_j^{*,k}} \le I_j^{k,+}, \\ \overline{w_j^{*,k}} - I_j^{k,+}, & I_j^{k,+} < \overline{w_j^{*,k}} \le 1, \end{cases} \ (k = 1, 2, ..., l; \ j = 1, 2, ..., m) & (23) \end{cases}$$

then, $r_{w^*}(x_h) = r^*(x_h) \quad (h = 1, 2, ..., g)$.

The proof of Lemma 1 is provided in Appendix B.

Lemma 2: For decision function with F the OWA operator as per Eq. (4), if there exists $\mathbf{w}^* = (w_1^*, w_2^*, ..., w_m^*)^T$ satisfying constraint conditions (17)-(25)

$$\begin{cases} \sum_{j=1}^{m} w_j^* \bar{v}_{i(j)} > \sum_{j=1}^{m} w_j^* \bar{v}_{h(j)} - (1 - y_{ih})M, \quad (i = 1, 2, ..., n; h = 1, 2, ..., g) & (24) \\ \sum_{j=1}^{m} w_j^* \bar{v}_{i(j)} \le \sum_{j=1}^{m} w_j^* \bar{v}_{h(j)} + y_{ih}M, \quad (i = 1, 2, ..., n; h = 1, 2, ..., g) & (25) \end{cases}$$

then, $r_{w^*}(x_h) = r^*(x_h) \quad (h = 1, 2, ..., g)$.

The proof of Lemma 2 is provided in Appendix B.

Based on Lemmas 1 and 2, we obtain the following Theorem 1.

Theorem 1: By introducing the transformed decision variables: $z_{jq}^k = 0 \ \ or \ \ 1$, $\sum_{q=1}^{3} z_{jq}^k = 1$, $(k = 1, 2, ..., l; \ q = 1, 2, 3; \ j = 1, 2, ..., m)$, we have

(1) If F is a WA operator, the MCSWM (model (14)) can be transformed into the mixed 0-1 linear programming (26)-(42).

$$\begin{cases} \min \sum_{k=1}^{l}\sum_{j=1}^{m} f_k[(I_j^{k,-}-\overline{w_j^k})z_{j1}^k+(\overline{w_j^k}-I_j^{k,+})z_{j3}^k] & (26) \\ s.t. \begin{cases} \sum_{j=1}^{m} w_j\bar{v}_{ij} > \sum_{j=1}^{m} w_j\bar{v}_{hj}-(1-y_{ih})M, \ (i=1,2,...,n; h=1,2,...,g) & (27) \\ \sum_{j=1}^{m} w_j\bar{v}_{ij} \le \sum_{j=1}^{m} w_j\bar{v}_{hj}+y_{ih}M, \ (i=1,2,...,n; h=1,2,...,g) & (28) \\ y_{ih}=1 \ or \ 0, \ (i=1,2,...,n; h=1,2,...,g) & (29) \\ \sum_{i=1}^{n} y_{ih}+1=r^*(x_h), \ (h=1,2,...,g) & (30) \\ w_j=\frac{\sum_{k=1}^{l}\overline{w_j^k}}{l}, \ (j=1,2,...,m) & (31) \\ \sum_{j=1}^{m}\overline{w_j^k}=1, \ (k=1,2,...,l) & (32) \\ 0\le \overline{w_j^k}\le 1, (k=1,2,...,l; j=1,2,...,m) & (33) \\ w_j^k=[I_j^{k,-},I_j^{k,+}], \ (k=1,2,...,l; j=1,2,...,m) & (34) \\ \overline{w_j^k}-I_j^{k,-}\le 0+(1-z_{j1}^k)M, \ (k=1,2,...,l; j=1,2,...,m) & (35) \\ \overline{w_j^k}\ge 0-(1-z_{j1}^k)M, \ (k=1,2,...,l; j=1,2,...,m) & (36) \\ \overline{w_j^k}-I_j^{k,-}> 0+(1-z_{j2}^k)M, \ (k=1,2...,l; j=1,2,...,m) & (37) \\ \overline{w_j^k}-I_j^{k,+}\le 0-(1-z_{j2}^k)M, \ (k=1,2...,l; j=1,2,...,m) & (38) \\ \overline{w_j^k}-I_j^{k,+}> 0+(1-z_{j3}^k)M, \ (k=1,2...,l; j=1,2,...,m) & (39) \\ \overline{w_j^k}-1\le 0-(1-z_{j3}^k)M, \ (k=1,2...,l; j=1,2,...,m) & (40) \\ z_{j1}^k+z_{j2}^k+z_{j3}^k=1, \ (k=1,2...,l; j=1,2,...,m) & (41) \\ z_{jq}^k=0 \ or \ 1, \ (q=1,2,3; k=1,2,...,l; j=1,2,...,m) & (42) \end{cases} \end{cases}$$

(2) In (26)-(42) above, substitute constraints (27)-(28) into constraints (43)-(44)

$$\begin{cases} \sum_{j=1}^{m} w_j\bar{v}_{i(j)} > \sum_{j=1}^{m} w_j\bar{v}_{h(j)}-(1-y_{ih})M, \ (i=1,2,...,n; h=1,2,...,g) & (43) \\ \sum_{j=1}^{m} w_j\bar{v}_{i(j)} \le \sum_{j=1}^{m} w_j\bar{v}_{h(j)}+y_{ih}M, \ (i=1,2,...,n; h=1,2,...,g) & (44) \end{cases}$$

If F is an OWA operator, the MCSWM (model (14)) can be transformed into the mixed 0-1 linear programming model (26), (29)-(44).

The proof of Theorem 1 is provided in Appendix B.

In this paper, denote models (26)-(42) as P_1, and models (26), (29)-(44) as P_2. In both P_1 and P_2 models, $\overline{w_j^k}$ $(k=1,2,...,l;$ $j=1,2,...,m)$; y_{ih} $(i=1,2,...,n;\ h=1,2,...,g)$; z_{jq}^k, $(q=1,2,3;$ $j=1,2,...,m)$ are the decision variables.

Based on Theorem 1, we can obtain the optimal solution to the MCSWM via mixed 0-1 linear programming. Clearly, if the optimal solution to the MCSWM exists, a manipulator can set a strategic weight vector to obtain his/her desired ranking of the alternatives $\{x_1,x_2,...,x_g\}$. Otherwise, it is not possible to strategically manipulate the attribute weights to achieve his/her goal.

C. *Some desired properties for models P_1 and P_2*

In this subsection, we present some desired properties of the MCSWM. In order to make the proposed properties easy to understand, we first introduce the ranking range of an alternative.

In a MADM problem, let $W=\{(w_1,w_2,...,w_m)^T \mid \sum_{j=1}^{m} w_j=1,\ 0\le w_j\le 1\}$ be the set of attribute weights without any constraint; $\underline{r}_{w\in W}(x_h)=\min_{w\in W} r_w(x_h)$ the best ranking of alternative x_h under W; and $\overline{r}_{w\in W}(x_h)=\max_{w\in W} r_w(x_h)$ the worst ranking of alternative x_h under W. Then, $R_{w\in W}(x_h)=[\underline{r}_{w\in W}(x_h),\overline{r}_{w\in W}(x_h)]$ is called the ranking range of alternative x_h under the set of attribute weights W.

Let $S=\{(w_1,w_2,...,w_m)^T \mid \sum_{j=1}^{m} w_j=1,\ 0\le w_j\le 1,\ w_j=\frac{\sum_{k=1}^{l} w_j^k}{l},\ 0\le w_j^k\le 1,$ $\sum_{j=1}^{m} w_j^k=1,\ w_j^k\in[I_j^{k,-},I_j^{k,+}]\}$ be the set of interval attribute weights; $\underline{r}_{w\in S}(x_h)=\min_{w\in S} r_w(x_h)$ the best ranking of alternative x_h under S; and $\overline{r}_{w\in S}(x_h)=\max_{w\in S} r_w(x_h)$ the worst ranking of alternative x_h under S. Then, $R_{w\in S}(x_h)=[\underline{r}_{w\in S}(x_h),\overline{r}_{w\in S}(x_h)]$ is called the ranking range of alternative x_h under the set of interval attribute weights S.

Specifically, when the WA operator F, as per Eq. (3), is used to compute the decision evaluation function, let $R_{w\in W}^{WA}(x_h)=[\underline{r}_{w\in W}^{WA}(x_h),\overline{r}_{w\in W}^{WA}(x_h)]$ and $R_{w\in S}^{WA}(x_h)=[\underline{r}_{w\in S}^{WA}(x_h),\overline{r}_{w\in S}^{WA}(x_h)]$ be the ranking range under W and S, respectively. When OWA operator F, as per Eq. (4), is used to compute the decision evaluation function, let $R_{w\in W}^{OWA}(x_h)=[\underline{r}_{w\in W}^{OWA}(x_h),\overline{r}_{w\in W}^{OWA}(x_h)]$ and $R_{w\in S}^{OWA}(x_h)=[\underline{r}_{w\in S}^{OWA}(x_h),\overline{r}_{w\in S}^{OWA}(x_h)]$ be the ranking range under W and S, respectively.

Then, the following three desired properties to manipulate the attribute weights are presented as Properties 1-3.

Property 1: For a desired ranking $\{r^*(x_1),r^*(x_2),...,r^*(x_g)\}$, we have that

(1) if the objective value of P_1 is zero, then $r^*(x_h)\in[\underline{r}_{w\in S}^{WA}(x_h),\overline{r}_{w\in S}^{WA}(x_h)],\ \forall h\in\{1,2,...,g\}$.

(2) if the objective value of P_2 is zero, then $r^*(x_h)\in[\underline{r}_{w\in S}^{OWA}(x_h),\overline{r}_{w\in S}^{OWA}(x_h)],\ \forall h\in\{1,2,...,g\}$.

The proof of Property 1 is provided in Appendix B.

Property 1 provides the necessary condition that make possible for a manipulator to manipulate a strategic attribute weight with zero cost to obtain a desired ranking of alternatives under the WA and OWA operators, respectively.

Property 2: When $g=1$, we have that

(1) if the objective value of P_1 is zero if and only if $r^*(x_h)\in[\underline{r}_{w\in S}^{WA}(x_h),\overline{r}_{w\in S}^{WA}(x_h)]$.

(2) if the objective value of P_2 is zero if and only if $r^*(x_h)\in[\underline{r}_{w\in S}^{OWA}(x_h),\overline{r}_{w\in S}^{OWA}(x_h)]$.

The proof of Property 2 is provided in Appendix B.

Property 2 provides the necessary and sufficient condition for a manipulator to manipulate a strategic attribute weight with zero cost to obtain any desired ranking of one alternative under the WA and OWA operators, respectively.

Property 3: For a desired ranking $\{r^*(x_1), r^*(x_2), ..., r^*(x_g)\}$, we have that

(1) the solution of model P_1 does not exist if it satisfies the condition $\exists\ h \in \{1,2,...,g\}$, $r^*(x_h) \notin [\underline{r}^{WA}_{w\in W}(x_h), \overline{r}^{WA}_{w\in W}(x_h)]$.

(2) the solution of model P_2 does not exist if it satisfies the condition $\exists\ h \in \{1,2,...,g\}$, $r^*(x_h) \notin [\underline{r}^{OWA}_{w\in W}(x_h), \overline{r}^{OWA}_{w\in W}(x_h)]$.

The proof of Property 3 is provided in Appendix B.

Property 3 provides the condition under which a manipulator cannot manipulate a strategic weight vector under any cost to obtain his/her desired ranking.

IV. Numerical Analysis and Simulation Experiments

In this section, we present an example with real data (provided in Appendix C) from the Academic Ranking of World Universities (ARWU; http://www.arwu.org/) [34] and several simulation experiments to show the validity and desired properties of the proposed minimum cost strategic weight manipulation model.

A. Numerical analysis

Let 50 Universities taken from ARWU be the set of alternatives $\{x_1, x_2, ..., x_{50}\}$, which will be ranked using the following set of 6 attributes $\{a_1, a_2, ..., a_6\}$:

a_1: Quality of Education (Alumni: Alumni of an institution winning Nobel Prizes and Fields Medals);

a_2: Quality of Faculty 1 (Award: Staff of an institution winning Nobel Prizes and Fields Medals);

a_3: Quality of Faculty 2 (HiCi: Highly Cited researchers in 21 broad subject categories);

a_4: Papers published in Nature and Science (N&S);

a_5: Papers indexed in Science Citation Index-expanded and Social Science Citation Index (PUB);

a_6 :Per capita academic performance of an institution (PCP).

First, we transform the data for the 50 universities regarding the set of attributes above into a normalized decision matrix $\overline{V} = [\overline{v}_{ij}]_{50\times 6}$. Let $E = \{e_1, e_2, e_3\}$ be a set of 3 experts. Let $\mathbf{w}^1 = (w_1^1, w_2^1, ..., w_6^1)^T$, where $w_1^1 = [0.1, 0.3]$, and $w_j^1 = [0,1]$, $j = 2,3,4,5,6$ be the interval attribute weights of expert e_1; $\mathbf{w}^2 = (w_1^2, w_2^2, ..., w_6^2)^T$, where $w_2^2 = [0.2, 0.6]$, and $w_j^2 = [0,1]$, $j = 1,3,4,5,6$ be the interval attribute weights of expert e_2; and $\mathbf{w}^3 = (w_1^3, w_2^3 ..., w_6^3)^T$, where $w_4^3 = [0.4, 0.8]$, $w_5^3 = [0.2, 0.9]$, and $w_j^3 = [0,1]$, $j = 1,2,3,6$ be the interval attribute weights of expert e_3.

Without loss of generality, let $f_k = 1,\ (k = 1,2,3)$, be the unit cost of revising decision maker's original attribute weights. In the following, we assume that an expert wants to manipulate the alternative x_h, and his/her desired ranking for such alternative is r^*. Then, based on models P_1 and P_2, the manipulator can strategically set an attribute weight vector w^* with minimum cost C^*, to obtain his/her desired goal of ranking. For example:

(1) Let x_3 be the manipulated alternative, and $r^*(x_3) = 3$ the corresponding desired ranking. If F is the WA operator, then this is possible as P_1 results in the following strategic attribute weight vector $\mathbf{w}^* = (0.367, 0.067, 0.033, 0.133, 0.4, 0)$ with minimum cost $C^* = 0$;

(2) Let x_{20} be the manipulated alternative, and $r^*(x_{20}) = 15$ the corresponding desired ranking. If F is the OWA operator, then this is possible as P_2 results in the following strategic weight vector $\mathbf{w}^* = (0.421, 0.375, 0, 0, 0.011, 0.194)$ with minimum cost $C^* = 0.57$;

(3) Let $\{x_8, x_{13}, x_{14}, x_{15}\}$ be the manipulated alternatives, and $r^* = \{46, 23, 24, 13\}$ their corresponding desired ranking. If F is WA operator, then this is possible with P_1 resulting in the following strategic weight vector $\mathbf{w}^* = (0.064, 0.081, 0, 0.006, 0.849, 0)$ with minimum cost $C^* = 0.382$;

(4) Let $\{x_9, x_{10}, x_{11}, x_{12}\}$ be the manipulated alternatives, and $r^* = \{19, 7, 27, 16\}$ their desired ranking. If F is OWA operator, then because there is no solution to P_2, the manipulator will be unable to strategically set an attribute weight vector to achieve the desired ranking.

Table 1 shows a strategic weight vector w^* with its corresponding minimum cost C^* for different manipulated alternative(s) x^* to achieve a desired ranking r^*.

Table 1: Strategic weight vector w^* with minimum cost C^* for different manipulated alternative(s) x^* and desired ranking r^*

WA			
Manipulated alternative(s)	r^*	$\mathbf{w}^*$	C^*
x_3	3	(0.37, 0.07, 0.03, 0.13, 0.4, 0)	0
x_6	10	(0.03, 0.06, 0, 0.13, 0.77, 0.03)	0.019
x_{20}	9	No solution	~
$\{x_8, x_{13}, x_{14}, x_{15}\}$	{2, 6, 10, 12}	No solution	~
	{46, 23, 24, 13}	(0.06, 0.08, 0, 0.01, 0.85, 0)	0.382
$\{x_9, x_{10}, x_{11}, x_{12}\}$	{6, 7, 8, 9}	No solution	~
	{6, 8, 9, 10}	(0.12, 0.2, 0, 0.13, 0.54, 0)	0
$\{x_{20}, x_{23}, x_{25}, x_{27}\}$	{13, 3, 16, 4}	(0, 0, 0.17, 0.01, 0.82, 0)	0.667
	{10, 2, 15, 3}	No solution	~
OWA			
Manipulated alternative(s)	r^*	$\mathbf{w}^*$	C^*
x_3	8	(0.98, 0.02, 0, 0, 0, 0)	1.37
x_6	6	(0.37, 0.13, 0.3, 0.13, 0.07, 0)	0
x_{20}	15	(0.42, 0.38, 0, 0, 0.01, 0.19)	0.57

$\{x_8,x_{13},x_{14},x_{15}\}$	$\{10,11,12,13\}$	No solution	~
	$\{6,12,13,14\}$	$(0.1,0.07,0.63,0.01,03,0.07)$	0.357
$\{x_9,x_{10},x_{11},x_{12}\}$	$\{8,9,10,11\}$	$(0.03,0.83,0,0.13,0.01,0)$	0.198
	$\{19,7,27,16\}$	No solution	~
$\{x_{20},x_{23},x_{25},x_{27}\}$	$\{17,9,21,10\}$	$(0.03,0.04,0.13,0.13,0.6,0.05)$	0.069
	$\{24,45,46,47\}$	No solution	~

From Table 1, it can be noticed that in some cases the manipulator incurred zero cost ($C^*=0$) to set a strategic weight vector to obtain his/her goal. On the other hand, in some other cases the manipulator is unable to set a strategic weight vector under any cost. In the following, we will verify the validity of the conditions presented in Properties 1-3. Table 2 shows the ranking ranges $R^{WA}_{w\in W}$, $R^{OWA}_{w\in W}$, $R^{WA}_{w\in S}$ and $R^{OWA}_{w\in S}$ for the 50 universities.

Table 2: Ranking ranges $R^{WA}_{w\in W}$, $R^{OWA}_{w\in W}$, $R^{WA}_{w\in S}$ and $R^{OWA}_{w\in S}$ for 50 universities

x_i	x_1	x_2	x_3	x_4	x_5	x_6
$R^{WA}_{w\in W}$	[1,2]	[2,11]	[2,13]	[2,14]	[2,25]	[2,47]
$R^{OWA}_{w\in W}$	[1,2]	[2,7]	[2,9]	[2,5]	[2,7]	[2,9]
$R^{WA}_{w\in S}$	[1,1]	[2,4]	[3,5]	[2,5]	[2,5]	[6,9]
$R^{OWA}_{w\in S}$	[1,1]	[2,2]	[4,5]	[4,5]	[3,3]	[6, 6]

x_i	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
$R^{WA}_{w\in W}$	[2,9]	[1,47]	[4,28]	[4,42]	[5,26]	[4,27]
$R^{OWA}_{w\in W}$	[5,11]	[1,9]	[6,17]	[6,12]	[8,24]	[10,19]
$R^{WA}_{w\in S}$	[7,9]	[6,11]	[6,8]	[8,11]	[8,11]	[12,15]
$R^{OWA}_{w\in S}$	[8,8]	[7,7]	[9,9]	[10,10]	[11,11]	[12,12]

x_i	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}
$R^{WA}_{w\in W}$	[9,28]	[5,36]	[5,36]	[7,33]	[5,30]	[9,28]
$R^{OWA}_{w\in W}$	[8,30]	[11,31]	[11,31]	[10,25]	[10,30]	[13,27]
$R^{WA}_{w\in S}$	[12,14]	[12,15]	[14,16]	[13,16]	[17,20]	[20,21]
$R^{OWA}_{w\in S}$	[13,13]	[14,14]	[17,22]	[15,15]	[16,20]	[16,21]

x_i	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}
$R^{WA}_{w\in W}$	[8,38]	[10,43]	[11,50]	[10,46]	[3,50]	[9,46]
$R^{OWA}_{w\in W}$	[13,36]	[15,34]	[15,50]	[18,43]	[9,50]	[15,46]
$R^{WA}_{w\in S}$	[17,19]	[17,20]	[19,21]	[25,31]	[22,33]	[22,29]
$R^{OWA}_{w\in S}$	[16,22]	[23,25]	[16,21]	[22,23]	[16,24]	[16,23]

x_i	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}
$R^{WA}_{w\in W}$	[5,49]	[8,45]	[2,48]	[15,48]	[22,50]	[9,47]
$R^{OWA}_{w\in W}$	[13,48]	[14,43]	[8,48]	[19,44]	[23,41]	[20,42]
$R^{WA}_{w\in S}$	[27,35]	[31,37]	[22,26]	[22,25]	[22,26]	[28,35]
$R^{OWA}_{w\in S}$	[26,34]	[24,25]	[26,34]	[26,29]	[27,31]	[26,29]

x_i	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}
$R^{WA}_{w\in W}$	[17,43]	[16,50]	[16,50]	[16,49]	[22,48]	[22,49]
$R^{OWA}_{w\in W}$	[17,43]	[21,43]	[20,41]	[22,48]	[27,43]	[25,49]
$R^{WA}_{w\in S}$	[27,31]	[25,31]	[33,39]	[35,41]	[38,49]	[36,43]
$R^{OWA}_{w\in S}$	[29,35]	[30,34]	[32,35]	[36,40]	[36,38]	[29,34]

x_i	x_{37}	x_{38}	x_{39}	x_{40}	x_{41}	x_{42}
$R^{WA}_{w\in W}$	[10,50]	[21,49]	[15,50]	[13,50]	[27,49]	[15,50]
$R^{OWA}_{w\in W}$	[20,48]	[22,49]	[24,49]	[18,50]	[33,50]	[24,50]
$R^{WA}_{w\in S}$	[23,40]	[32,38]	[29,36]	[44,49]	[41,44]	[45,49]
$R^{OWA}_{w\in S}$	[38,43]	[27,35]	[38,42]	[41,48]	[45,48]	[36,42]

x_i	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}
$R^{WA}_{w\in W}$	[22,50]	[13,50]	[28,50]	[11,50]	[21,50]	[15,50]
$R^{OWA}_{w\in W}$	[26,50]	[23,50]	[37,50]	[22,50]	[29,49]	[24,49]
$R^{WA}_{w\in S}$	[37,45]	[45,47]	[37,42]	[30,44]	[42,44]	[46,49]
$R^{OWA}_{w\in S}$	[40,48]	[40,44]	[47,49]	[36,42]	[46,49]	[42,44]

x_i	x_{49}	x_{50}
$R^{WA}_{w\in W}$	[27,50]	[30,50]
$R^{OWA}_{w\in W}$	[38,49]	[30,50]
$R^{WA}_{w\in S}$	[47,50]	[49,50]
$R^{OWA}_{w\in S}$	[45,49]	[50,50]

Based on the data from Tables 1 and 2, we find the results to be consistent with Properties 1-3.

B. *Numerical analysis*

In this subsection, we present simulation experiments to analyse the effect the interval attribute weights and the unit cost have on the MCSWM.

(1) *The effect of interval attribute weights*

First, we consider the constraints for the attribute weights. Let $S_j=\{(w_1,...,w_j,...,w_m)^T \mid w_i\in[I_i^-,I_i^+]\}$ $(j=1,2,...,m)$ be a set of interval attribute weights, where $[I_i^-,I_i^+]\subset[0,1]$ $(i\le j)$ and $[I_i^-,I_i^+]=[0,1]$ $(i>j)$. In the other words, set S_j constraints only the weight of an attribute a_i with $i\le j$. In Simulation Experiment I below, set S_j is randomly generated, and thus the bigger the value j the more constraints on attribute weights there are, in the sense of average cases.

Let $r^*(x_h)$ be the manipulator's desired ranking of the alternative x_h, and f_k $(k=1,2,...,l)$ the unit cost to revise the expert e_k's original interval attribute weights. Let $F^{WA}_{S_j}(x_h)$ and $F^{OWA}_{S_j}(x_h)$ be the minimum cost to find out a strategic weight vector from the set S_j to obtain the manipulator's desired goal ranking of alternative x_h under the WA and the OWA operators, respectively.

Next, we design Simulation Experiment I to analyse the effect of interval attribute weights on the minimum cost to manipulate a strategic weight vector. Without loss of generality, we set $f_k=1,$ $(k=1,2,...,l)$ and set the manipulated alternative to be x_1.

Simulation Experiment I:

Input: n,m and j

Output: $F^{WA}_{S_j}(x_1)$ and $F^{OWA}_{S_j}(x_1)$

Step 1: *(generation of the standardized decision matrix)*: Generate randomly a standardized decision matrix $\bar{V}=[\bar{v}_{ij}]_{n\times m}$, where $\bar{v}_{ij}\in[0,1]$.

Step 2: *(generation of the desired ranking of the alternative x_1)*: Apply methods from Dong et al. [12] to obtain the ranking ranges of the alternative x_1, $\left[\underline{r}^{WA}_{w\in W}(x_1), \overline{r}^{WA}_{w\in W}(x_1)\right]$ and $\left[\underline{r}^{OWA}_{w\in W}(x_1), \overline{r}^{OWA}_{w\in W}(x_1)\right]$, for the WA and the OWA operators, respectively. Let $r^*(x_1)$ be the manipulator's desired ranking of the alternative x_1. When using the WA operator, the value of $r^*(x_1)$ is randomly selected from $\left[\underline{r}^{WA}_{w\in W}(x_1), \overline{r}^{WA}_{w\in W}(x_1)\right]$. When using the OWA operator, the value of $r^*(x_1)$ is randomly selected from $\left[\underline{r}^{OWA}_{w\in W}(x_1), \overline{r}^{OWA}_{w\in W}(x_1)\right]$.

Step 3: *(generation of the interval attribute weights sets S_j)*: Generate randomly a set of interval attribute weights $S_j = \{(w_1,...,w_j,...,w_m)^T \mid w_i \in [I_i^-, I_i^+]\}$: generate a random integer number j from set $\{1,2,...,m\}$; generate random values I_i^- $(i \le j)$ and I_i^+ $(i \le j)$ from $[0,1)$ and $[I_i^-, 1)$, respectively, and set $[I_i^-, I_i^+] = [0,1]$ $(i > j)$. Apply models P_1 and P_2 to obtain $F^{WA}_{S_j}(x_1)$ and $F^{OWA}_{S_j}(x_1)$, respectively. Compute the minimum cost $F^{WA}_{S_j}(x_1)$ and $F^{OWA}_{S_j}(x_1)$ to find out a strategic weight vector from the set S_j to obtain the manipulator's desired goal ranking of alternative x_1 under the WA and the OWA operators, respectively.

We set different values of n , m and j , and run 100 times Simulation Experiment I to obtain average values of $F^{WA}_{S_j}(x_1)$ and $F^{OWA}_{S_j}(x_1)$, which are shown in Fig.1 below.

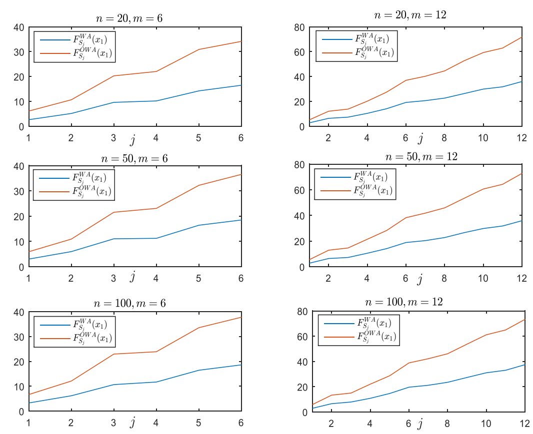

Fig. 1. Average values of $F^{WA}_{S_j}(x_1)$ and $F^{OWA}_{S_j}(x_1)$ under different parameters in Simulation Experiments I.

Clearly, Fig. 1 shows that: (i) in all cases, the average minimum cost to set strategic weight vectors under the WA operator is smaller than that under the OWA operator; and (ii) the average minimum cost to set strategic weight vectors under the OWA operator increase more quickly than that under the WA operator, as the attribute weights constraints increase.

(2) The effect of the unit cost

In Simulation Experiment II, we assumed that the unit cost to revise the original attribute weights information is the same for all experts, i.e. $f_k = f$, $(k = 1,2,...,l)$, and respectively set as $f = \frac{u}{100}$ $(u = 1,2,...,100)$ to study the effect of unit cost on the minimum cost to strategically manipulate the attribute weight vector in the MCSWM. Clearly, the larger the value of u , the higher the unit cost is.

Let $r^*(x_h)$ be the manipulator's desired ranking of the alternative x_h. When setting $f = \frac{u}{100}$, let $F^{WA,u}_S(x_1)$ be the minimum cost to find out a strategic weight vector from a set of interval attribute weights S to achieve the manipulator's desired goal ranking of alternative x_h under the WA and the OWA operators, respectively. Without loss of generality, we set the manipulated alternative to be x_1 and the sets of interval attribute weights are generated randomly.

Simulation Experiment II:

Input: n, m and u

Output: $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$

Step 1: Same to Step 1 in Simulation Experiment I.

Step 2: Same to Step 2 in Simulation Experiment I.

Step 3: *(generation of the interval attribute weights sets S)*: Generate the set of interval attribute weights $S = \{(w_1, w_2, ..., w_m)^T \mid w_j \in [I_j^-, I_j^+],\ j = 1,2,...,m\}$ by randomly selecting I_j^- and I_j^+ from $[0,1)$ and $[I_j^-, 1)$, respectively.

Step 4: *(calculation of the minimum cost $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$)*: Set $f_k = f$, $(k = 1,2,...,l)$ and $f = \frac{u}{100}$. Apply models P_1 and P_2 to obtain $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$, respectively. Compute the minimum cost $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$ to find out a strategic weight vector from the set S to obtain the manipulator's desired goal ranking of alternative x_1 under the WA and the OWA operators, respectively.

We set different values of n, m and u, and run 100 times Simulation Experiment II to obtain average values of $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$, which are depicted in Fig.2.

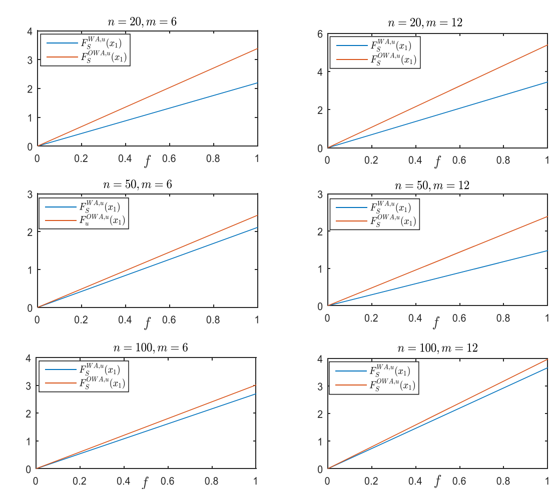

Fig. 2. Average values of $F^{WA,u}_S(x_1)$ and $F^{OWA,u}_S(x_1)$ under different parameters in Simulation Method II.

As with Simulation Method I, it is evident from Fig.2 that: (i) in all cases the average minimum cost to set strategic weight vectors under the OWA operator is larger than that under the WA operator; and (ii) as the unit cost increases, the average minimum cost to set strategic weight vectors under the OWA operator increases more quickly than that under the WA operator.

Simulation Methods I and II both show a better performance of the OWA operator than the WA operator in defending against the strategic weight manipulation of the MADM problems because of the higher associated minimum cost. Furthermore, as the attribute weights constraints and the unit cost increase, the performance of the OWA operator as a defense mechanism against the strategic weight manipulation increases faster than if the WA operator were used instead.

Consequently, it can be concluded that the OWA operator provides a better defense mechanism than the WA operator against multiple attribute strategic weight manipulation with interval attribute weights information.

V. Conclusions

This paper focuses on the strategic weight manipulation with minimum cost to obtain a desired ranking of alternatives, in a group decision context with interval attribute weights information. The existing approaches to set attribute weights have been investigated intensively, however, in these approaches decision makers are assumed to be honest aiming to obtain "best" attribute weights to get a ranking of alternatives. This paper follows the new assumption presented in [12] that the decision makers are not honest to strategically set attribute weights to obtain their desired ranking of the alternatives. The main contributions presented in this paper are:

(1) The strategic weight manipulation issue in [12] was investigated in an individual decision making context with no constraints on the attribute. In this paper, we present the minimum cost strategic weight manipulation model in a group decision context with interval attribute weights information.

(2) We discuss the conditions based on the ranking range under interval attribute weights information for (i) the existence of a weight vector to be set strategically to achieve the manipulator's desired ranking, and (ii) zero cost for the manipulation.

(3) We present detailed simulation experiments to reveal the effects of the attribute weights information and the unit cost on the minimum cost to manipulate strategic weights in a group context.

Meanwhile, we argue that it will be an interesting future research topic to investigate multiple attribute strategic weight manipulation in a consensus-reaching context [14, 49, 50] and the presence of trust relationship [39, 40].

Appendix A. Notations

The main notations in this paper are as follows.

X : The set of alternatives;

A : The set of attributes;

E : The set of experts;

$V=[v_{ij}]_{n\times m}$: Decision matrix;

$\bar{V}=[\bar{v}_{ij}]_{n\times m}$: Standardized decision matrix;

$D_w(x_i)$: The evaluation function of alternative x_i with weight vector w;

$\mathbf{w}$: The weight vector of attribute weights;

$\mathbf{w}^{\mathbf{k}}$: The original attribute weight vector over expert e_k;

$\overline{\mathbf{w}^{k}}$: The revised attribute weight vector associated with expert e_k;

$r_w(x_h)$: The ranking of alternative x_h under attribute weight vector $\mathbf{w}$;

W : The attribute weights set without any constraint;

$\underline{r}_{w\in W}(x_h)$: The best ranking of alternative x_h under the set of attribute weights W;

$\overline{r}_{w\in W}(x_h)$: The worst ranking of alternative x_h under the set of attribute weights W;

$R_{w\in W}(x_h)=[\underline{r}_{w\in W}(x_h),\overline{r}_{w\in W}(x_h)]$: Ranking range under the set of attribute weights W;

$R^{WA}_{w\in W}(x_h)=[\underline{r}^{WA}_{w\in W}(x_h),\overline{r}^{WA}_{w\in W}(x_h)]$: Ranking range under the set of attribute weights W associated with the WA operator;

$R^{OWA}_{w\in W}(x_h)=[\underline{r}^{OWA}_{w\in W}(x_h),\overline{r}^{OWA}_{w\in W}(x_h)]$: Ranking range under the set of attribute weights W associated with the OWA operator;

S : The set of interval information of attribute weights;

$\underline{r}_{w\in S}(x_h)$: The best ranking of alternative x_h under the set of interval attribute weights S;

$\overline{r}_{w\in S}(x_h)$: The worst ranking of alternative x_h under the set of interval attribute weights S;

$R^{WA}_{w\in S}(x_h)=[\underline{r}^{WA}_{w\in S}(x_h),\overline{r}^{WA}_{w\in S}(x_h)]$: Ranking range under the set of interval attribute weights S associated with the WA operator;

$R^{OWA}_{w\in S}(x_h)=[\underline{r}^{OWA}_{w\in S}(x_h),\overline{r}^{OWA}_{w\in S}(x_h)]$: Ranking range under the set of interval attribute weights S associated with the OWA operator;

Appendix B. Proofs

Proof of Lemma 1:

(1) From the condition of $\mathbf{w}^*=(w_1^*,w_2^*,...,w_m^*)$ verifying the constraints (15)-(23),

$$\begin{cases}\sum_{j=1}^{m} w_j^* \overline{v}_{ij} > \sum_{j=1}^{m} w_j^* \overline{v}_{hj} - (1-y_{ih})M, \ (i=1,2,...,n; h=1,2,...,g) \\ \sum_{j=1}^{m} w_j^* \overline{v}_{ij} \le \sum_{j=1}^{m} w_j^* \overline{v}_{hj} + y_{ih}M, \ (i=1,2,...,n; h=1,2,...,g) \\ y_{ih}=1 \ \ or \ \ 0, \ (h=1,2,...,g) \\ \sum_{i=1}^{n} y_{ih} + 1 = r^*(x_h), \ (i=1,2,...,n; \ \ h=1,2,...,g) \\ w_j^* = \frac{\sum_{k=1}^{l} \overline{w_j^{*,k}}}{l}, \ (j=1,2,...,m) \\ \sum_{j=1}^{m} \overline{w_j^{*,k}} = 1, \ (k=1,2,...,l) \\ 0 \le \overline{w_j^{*,k}} \le 1, \ (j=1,2,...,m) \\ w_j^k = [I_j^{k,-}, I_j^{k,+}], \ (k=1,2,...,l; j=1,2,...,m) \\ d(\overline{w_j^{*,k}}, w_j^k) = \begin{cases} I_j^{k,-} - \overline{w_j^{*,k}}, & 0 \le \overline{w_j^{*,k}} \le I_j^{k,-} \\ 0, & I_j^{k,-} < \overline{w_j^{*,k}} \le I_j^{k,+} \\ \overline{w_j^{*,k}} - I_j^{k,+}, & I_j^{k,+} < \overline{w_j^{*,k}} \le 1 \end{cases} \ (k=1,2,...,l; \\ j=1,2,...,m) \end{cases}$$

Substitute $y_{ih}=1$ into the first and second constraints, then $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} > \sum_{j=1}^{m} w_j^* \overline{v}_{hj}$ and $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} \le \sum_{j=1}^{m} w_j^* \overline{v}_{hj} + 1 \cdot M \quad (h=1,2,...,g)$ can be obtained. According to the result (1) in subsection 3.1, $x_i \succ x_h \quad (h=1,2,...,g)$ can be guaranteed. If $y_{ih}=0$, then $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} \le \sum_{j=1}^{m} w_j^* \overline{v}_{hj}$ and $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} > \sum_{j=1}^{m} w_j^* \overline{v}_{hj} - 1 \cdot M \quad (h=1,2,...,g)$. According to result (2) in subsection 3.1, $x_i \precsim x_h \quad (h=1,2,...,g)$ can be guaranteed. The strategic weight vector w^* can be obtained by revising the decision maker's original attribute weights, so the distance $d(\overline{w_j^{*,k}}, w_j^k)$ should be given. Due to the non-negative property of distance functions, we can obtain different distance formula for the different ranges of $\mathbf{w}^*$, i.e.

$$d(\overline{w_j^{*,k}}, w_j^k) = \begin{cases} I_j^{k,-} - \overline{w_j^{*,k}}, & 0 \le \overline{w_j^{*,k}} \le I_j^{k,-} \\ 0, & I_j^{k,-} < \overline{w_j^{*,k}} \le I_j^{k,+} \\ \overline{w_j^{*,k}} - I_j^{k,+}, & I_j^{k,+} < \overline{w_j^{*,k}} \le 1 \end{cases}.$$

Finally, the constraint condition $\sum_{i=1}^{n} y_{ih} + 1 = r^*(x_h), \ (i=1,2,..., \ n; \ h=1,2,...,g)$ can guarantee $r_{w^*}(x_h) = r^*(x_h) \quad (h=1,2,...,g)$.

This completes the proof of Lemma 1.

Proof of Lemma 2:

Substitute the WA operator $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} > \sum_{j=1}^{m} w_j^* \overline{v}_{hj} - (1-y_{ih})M$ and $\sum_{j=1}^{m} w_j^* \overline{v}_{ij} \le \sum_{j=1}^{m} w_j^* \overline{v}_{hj} + y_{ih}M$ into the OWA operator $\sum_{j=1}^{m} w_j^* \overline{v}_{i(j)} > \sum_{j=1}^{m} w_j^* \overline{v}_{h(j)} - (1-y_{ih})M$ and $\sum_{j=1}^{m} w_j^* \overline{v}_{i(j)} \le \sum_{j=1}^{m} w_j^* \overline{v}_{h(j)} + y_{ih}M$ in proof of Lemma 1 and conclude that $r_{w^*}(x_h) = r^*(x_h)$.

This completes the proof of Lemma 2.

Proof of Theorem 1:

Introduce the following transformed decision variables z_{jq}^k, with $z_{jq}^k = 0$ or 1 $(q=1,2,3)$, $\sum_{q=1}^{3} z_{jq}^k = 1, \ (k=1,2,...,l; \ j=1,2,...,m)$.

Because $w_j^k = [I_j^{k,-}, I_j^{k,+}]$,

$$d\left(w_j^k, \overline{w_j^k}\right) = \begin{cases} I_j^{k,-} - \overline{w_j^k}, & 0 \le \overline{w_j^k} < I_j^{k,-}, \\ 0, & I_j^{k,-} \le \overline{w_j^k} \le I_j^{k,+}, \\ \overline{w_j^k} - I_j^{k,+}, & I_j^{k,+} < \overline{w_j^k} \le 1, \end{cases}$$

then, the mix 0-1 formulas $\overline{w_j^k} - I_j^{k,-} \le 0 + (1-z_{j1}^k)M$ and $\overline{w_j^k} \ge 0 - (1-z_{j1}^k)M$ guarantee $0 \le \overline{w_j^k} < I_j^{k,-}$;

$\overline{w_j^k} - I_j^{k,-} > 0 + (1-z_{j2}^k)M$ and $\overline{w_j^k} - I_j^{k,+} \le \ 0 - (1-z_{j2}^k)M$ guarantee $I_j^{k,-} \le \overline{w_j^k} \le I_j^{k,+}$;

$\overline{w_j^k} - I_j^{k,+} > (1-z_{j3}^k) \quad M$ and $\overline{w_j^k} - 1 \le 0 - (1-z_{j3}^k)M$ guarantee $I_j^{k,+} < \overline{w_j^k} \le 1$.

Then, we have $d\left(w_j^k, \overline{w_j^k}\right) = (I_j^{k,-} - \overline{w_j^k})z_{j1}^k + 0z_{j2}^k + (\overline{w_j^k} - I_j^{k,+})z_{j3}^k$.

According to Lemmas 1 and 2, plug models (15)-(23) and (17)-(25) into model (14) and transform the optimization models into the mixed 0-1 linear programming models (26)-(42) and (26), (29)-(44), respectively.

This completes the proof of Theorem 1.

Proof of Property 1:

Assuming that $\forall \ h \in \{1,2,...,g\}$, $r^*(x_h) \in [\underline{r}_{w\in S}^{WA}(x_h), \overline{r}_{w\in S}^{WA}(x_h)]$, the objective value of P_1 is non-zero, which means the manipulator must take some cost to revise the decision maker's original attribute weights. However, the condition $r^*(x_h) \in [\underline{r}_{w\in S}^{WA}(x_h), \ \overline{r}_{w\in S}^{WA}(x_h)]$ means the manipulator can obtain his/her ranking in the ranking range under the original interval attribute weights information. The above two results are in contradiction. This completes the proof of Property 1.

Proof of Property 2:

Sufficiency: When $g=1$, $r^*(x_h) \in [\underline{r}_{w\in S}^{WA}(x_h), \overline{r}_{w\in S}^{WA}(x_h)]$ means the manipulator can obtain his/her ranking in the ranking range under the original interval attribute weights information, it is evident that the objective value of P_1 is zero.

Necessity: The proof is same to the proof of Property 1.

Similarly, we can prove the property of model P_2.

Then, this completes the proof of Property 2.

Proof of Property 3:

Assuming that $\exists \ h \in \{1,2,...,g\}$, $r^*(x_h) \notin [\underline{r}_{w\in W}^{WA}(x_h), \overline{r}_{w\in W}^{WA}(x_h)]$, solutions of models P_1 and P_2 exist, which means that a manipulator can set strategic weight to achieve his/her desired ranking. However, according to the definition of ranking range under attribute weights W, $R^{WA}_{w\in W}(x_h)=[\underline{r}^{WA}_{w\in W}(x_h),\overline{r}^{WA}_{w\in W}(x_h)]$ means the ranking of alternative manipulator x_h vary in this range. The above two results are in contradiction. This completes the proof of Property 3.

APPENDIX C. THE ORIGINAL DATA FOR 50 UNIVERSITIES

x_i	v_{i1}	v_{i2}	v_{i3}	v_{i4}	v_{i5}	v_{i6}
1	100	100	100	100	100	79.2
2	42.9	89.6	80.1	73.6	73.1	55.8
3	65.1	79.4	64.9	68.7	68.4	59
4	78.3	96.6	51.3	56.7	67.8	58.5
5	69.4	80.7	55.3	71.7	61.7	69.7
6	53.3	98	51.3	47.2	42.9	74.4
7	49.7	54.9	56.2	55	74.5	46.1
8	51	66.7	39.7	57.3	43.6	100
9	63.5	65.9	41	53.3	68.9	33.3
10	59.8	86.3	34	42.7	50.2	44.5
11	47.6	50.4	44.7	58.4	62.6	37.1
12	29.5	47.1	58	44.5	71.4	33.4
13	42	49.8	41	47	60.5	40.9
14	19.2	35.5	49.2	57.8	63.5	37
15	21.2	31.6	49.2	52.1	72.6	31
16	37.7	33.6	38.4	47	71.9	31.1
17	28.1	36.2	41	41.6	73.9	32.4
18	31.6	33.8	42.3	39.4	67.7	37.8
19	29.5	35.5	35.5	50.2	55.6	46.1
20	36.3	25.3	30.8	47.5	70	29.7
21	0	39.9	37	52.1	59.3	33.5
22	14.5	35.8	43.5	32.9	64	39.9
23	34.4	0	51.3	41.6	76.6	25.8
24	34.4	24.9	51.3	42	51.7	37.2
25	15.4	19.2	57.1	38.9	62.1	25.9
26	15.4	22.1	54.3	35.6	59.6	32.8
27	19.9	17.2	32.4	38.2	80.1	30.3
28	32.8	34.8	30.8	35	62.7	24.3
29	28.1	31.9	32.4	39.5	57.3	22
30	21.8	18.8	32.4	36.2	65.2	41.9
31	29.9	36.2	30.8	33.1	55.1	29.1
32	31.6	37.2	27.1	31.5	58.4	23.8
33	29.5	16.3	39.7	32.5	64.8	24.1
34	15.4	18.8	42.3	32.7	64.5	27.2
35	18.5	32.6	37	26.4	58.4	29
36	8.9	23.7	39.7	32.6	60.8	33.8
37	17	59.8	27.1	41.8	19.3	40
38	12.6	34.1	30.8	36.8	46.2	35.1
39	33.6	27.4	20.5	29.7	61.9	25.3
40	17	13.3	35.5	24.8	67.9	32.2
41	20.5	24.9	32.4	31.3	52.1	26.8
42	14.5	39.1	32.4	27.3	37.7	38.2
43	18.5	34.5	30.8	37.6	34.9	27.7
44	25.6	26.6	22.9	25.1	52.6	40.2
45	16.2	16.3	29	37	56.3	26.6
46	30.3	54.3	10.3	17.6	47.9	27.7
47	19.9	25.3	22.9	30.6	51.8	34.9
48	34.8	21.6	29	23.3	49.7	34.6
49	0	31.7	35.5	23.4	53.9	26.2
50	21.2	21	34	19.6	55.3	27.9

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